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# THE INFLUENCE OF INACCURACY OF CALCULATING ALGORITHMS USED IN THE CMMS ON MEASUREMENT RESULTS -FINAL REPORT 


#### Abstract

The author has devised a test for evaluating the accuracy of calculation algorithms and used it to establish the accuracy of the software of fourteen CMMs offered by world top manufacturers. Using a minimum number of measuring points, the author calculated various geometrical elements with arbitrary shape deviations and the relationships between them, and compared the obtained results with the reference results. The input data were entered from the keyboard to the computer working with a CMM. Therefore the error of the final calculation result was introduced only by the software and the computer, and did not include the errors of the CMM. The computer and the software were treated as a "black box". The same test set was used for all software.


Keywords: coordinate measuring technique, evaluation software, computer simulation

## 1. INTRODUCTION

From time to time, every Coordinate Measuring Machine (CMM) should be calibrated (with complex or analytical methods) to establish its accuracy. During calibration we usually assume that the next element in the measuring system, the computer or, strictly speaking, the calculating algorithm, is not a source of additional errors. It seems, however, that it is an unfounded assumption and that the so-called "software quality" should be established before the calibration of the CMM is performed. The standard DIN 32880 [13] unequivocally defines the way of determining basic elements and the relationship between them. However, almost all CMM manufacturers use different software for control and for processing of input data. The author aims to show that with the CMM software used at present there is a considerable chance to obtain results with relatively large errors, especially when the number of measuring points is low. The author devised his own test for CMM software. The test allows every user to test his software and draw his own conclusions. The author used his test to examine software in industrial conditions. The measurement systems were not the author's property. Therefore it was necessary to work out a method which would require neither purchasing the software to be tested, nor the installation of sophisticated reference software. The author financed the research himself. In the test the substitute elements were calculated using a small number of measuring points because of limited access to the tested software. Besides, in this case the differences in the results are visible. To achieve a more detailed picture of software accuracy, the same set was tested for more points (up to 32 points). The results obtained in this part of the experiment, however, are beyond the scope of the presented paper. The tested software came from firms which have been operating on the Polish market since 1997 (Table 1).

Table 1. List of the tested CMM software (in alphabetical order).

| CMMs software | Comment |
| :--- | :--- |
| Calypso | $*$ |
| Geopak-Cosmos | $*$ |
| Metromeasure | $*$ |


| Metrosoft | $*$ |
| :--- | :--- |
| PC-DMIS | $* *$ |
| Prelude Inspection | $*$ |
| Quindos | $*$ For DOS |
| Quindos | *For WOS |
| Softios | $* *$ For Windows |
| Softios | $*$ |
| Tutor | $*$ |
| Umess | $* *$ New version |
| Zettmes |  |
| Zettmes |  |

*CMM software tested to 2002
** CMM software tested in 2005

## 2. DESCRIPTION OF THE TEST

In order to determine the value of errors generated by the measuring software, the author proposes a method of calibrating the measuring software with a 3D geometrical model standard. This is a virtual standard, which takes the form of a set of arbitrary points. Therefore the errors which are determined depend only on the type of computer and on the calculation algorithms. The measurement of a circle, plane and straight line is simulated with the minimum number of measuring points. Whether the small number of points would affect the accuracy of the real measurement is of secondary importance here. The obtained results are compared with the reference results, which were calculated with the Least Mean Square (LMS) method in accordance with the standard DIN 32880-1. In spite of the wellknown deficiencies of this method, it is the reference point of the whole test. All differences between the reference results and the obtained results are treated as calculation errors. In this way, the author receives a true image of the accuracy of the software tested. To avoid prejudice, the names of the manufacturers have been encoded. The letters A to O were used instead of company names.

The test has been conducted on the basis of elements of the ISO 10360-6 standard. The only difference is that the input data are entered in the computer through the keyboard. This allows the operator to introduce his own reference data instead of using commercial reference software. Moreover, since the CMM is not working while the test is conducted, it does not influence the measurement results. (Fig. 1).

The virtual 3D geometrical model comprises the most common geometrical elements with arbitrary shape deviations. This allows creating the measuring problem in an arbitrary way. The operator chooses the number and the distribution of measuring points. The points are not collected in a real measurement, but exist in virtual form. Only the computer and the software, therefore, introduce an error to the final measurement result.

The proposed approach is extremely cheap and flexible. It allows testing various types of CMM software. We may determine how a given piece of software calculates a straight line, a plane, a circle, a sphere, a cylinder, and how it calculates the relationships between these elements. The test involves typical measurement tasks without the so-called "exceptional cases". If we calculate a virtual geometrical element and compare the result with the result given by the reference software, we get reliable information about the real accuracy level of the software we use (Fig. 2).

Reference calculations were executed on the PC computer (Pentium III processor, Windows XP). Technical parameters of the tested computer and reference computer were comparable. The test has been carried out for 14 different cases of software. To eliminate mistakes, all tests were performed twice. In doubtful cases, the whole starting procedure was repeated, including the resetting of the computer.


Fig. 1. The block scheme of the Coordinate Measuring Technique.


Fig. 2. The block scheme of the comparison algorithm.
The experiment was conducted in two phases. In the first phase we tested the conformity of defining plane figures and solids in various pieces of software with the criterion of the minimum number of measuring points. In the second phase we attempted to determine the accuracy of particular algorithms.

Phase 1: The surface of almost all measured objects can be described by typical geometrical elements: a point, straight line, plane, circle, sphere, cylinder and cone. To determine all the above elements (which are parts of a measured object) we use the coordinates of points which belong to these elements. The coordinates of the points are either calculated or measured. For each element the smallest number of necessary points is defined. This is different, depending on whether the element is determined mathematically (fewer points are necessary) or whether it is calculated on the basis of measurements (more points are needed). Some tested software offers the option of using the mathematical minimum number of points for calculations. It seems pointless, however, since many more points are needed to reliably recreate a given shape [11]. Our test has shown that only one calculating algorithm fulfils the requirement of the minimum measurement number of
points for all elements. On the other hand, as many as three algorithms do not fulfil this requirement for all six geometrical elements. The algorithms use the minimal mathematical number of points instead.

Phase 2: The author has created a 3D geometrical model to be used in the tests. For the purpose of the test the author has selected measurement tasks which are most common in mechanical engineering. The simulated shape deviations are in the $6 \div 8$ accuracy class.
The test includes the determination of the following equations and relationships (Fig. 3):

1. equation of circle I ( no shape deviations),
2. equation of circle II (three-lobing),
3. coaxiality of circles I and II (Fig. 6a),
4. equation of circle III ( ovality),
5. equation of circle IV ( local flatness ),
6. distance between circles III and IV (Fig. 6b),
7. distance between planes I and II ( shape deviation of plane I- warp; of plane II concavity) (Fig. 6c),
8. angle between planes I and III ( shape deviation of plane I - warp; ideal plane III) (Fig. 6d),
9. intersection point of straight line and plane II ( shape deviation of plane II - concavity) (Fig. 6e).
The coordinates of points belonging to particular geometrical elements with deviations (in the $6 \div 8$ accuracy class) are entered in the computer working with the CMM. The final result shows only the errors generated by the computer and its software. The computer and software are treated as a black box. The same test is used in all 14 cases. The minimum measurement number of points is used for all geometrical elements. The distribution of particular geometrical elements in the 3D geometrical model is shown in Figs. 4-6, and the coordinates of these elements are given in Tables 2 to 9 .


Fig. 3. Distribution of geometrical elements in 3D space.
In measurements, the most common geometrical element is a circle (the shaft-hole element). Therefore in the 3D geometrical model there is one ideal circle and three circles with different shape deviations (three-lobing, ovality and local flatness). The angle distribution of the measuring points is the same for all four circles (Fig. 4a). The distribution of four measuring points for a given shape deviation is presented in Fig. 4b, Fig. 4c and Fig. 4d. Circles I and II are situated in plane XY, and circles III and IV lie in plane XZ.


Fig. 4. The distribution of measuring points in circles I-IV: a) close to ideal, b) three-lobing, c) ovality, d) flatness.
a)

c)

b)

d)


Fig. 5. The distribution of measuring points in: a) plane I, b) plane II, c) plane III, d) line.

Table 2. The coordinates of measuring points of circle I [mm]

|  | Point |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\#!$ | $\# 2$ | $\# 3$ | $\# 4$ |
| X | 65.321 | 37.144 | 32.679 | 64.142 |
| Y | 52.856 | 55.321 | 30 | 25.855 |
| Z | 0 | 0 | 0 | 0 |

Table 3. The coordinates of measuring points of circle II [mm]

|  | Point |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | $\# 1$ | $\# 2$ | $\# 3$ | $\# 4$ |


| X | 65.319 | 37.148 | 32.689 | 64.136 |
| :---: | :---: | :---: | :---: | :---: |
| Y | 52.852 | 55.318 | 30.009 | 25.862 |
| Z | 100 | 100 | 100 | 100 |

Table 4. The coordinates of measuring points of circle III [mm]

|  | Point |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\# 1$ | $\# 2$ | $\# 3$ | $\# 4$ |
| X | 57.658 | 43.564 | 41.337 | 57.065 |
| Y | 80 | 80 | 80 | 80 |
| Z | 77.660 | 77.658 | 65 | 62.923 |

Table 5. The coordinates of measuring points of circle IV [mm]

|  | Point |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\# 1$ | $\# 2$ | $\# 3$ | $\# 4$ |
| X | 57.660 | 43.565 | 41.339 | 57.071 |
| Y | 100 | 100 | 100 | 100 |
| Z | 36.428 | 37.658 | 25 | 22.929 |

Table 6. The coordinates of measuring points of plane I [mm]

|  | Point |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\# 1$ | $\# 2$ | $\# 3$ | $\# 4$ |
| X | 100 | 100 | 99.995 | 100 |
| Y | 70 | 75 | 10 | 15 |
| Z | 80 | 10 | 10 | 85 |

Table 7. The coordinates of measuring points of plane II [mm]

|  | Point |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\# 1$ | $\# 2$ | $\# 3$ | $\# 4$ |
| X | 0.005 | 0.002 | 0.004 | 0.003 |
| Y | 75 | 75 | 10 | 15 |
| Z | 80 | 10 | 10 | 85 |

Table 8. The coordinates of measuring points of plane III [mm]

|  | Point |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\# 1$ | $\# 2$ | $\# 3$ | $\# 4$ |
| X | 90 | 15 | 10 | 80 |
| Y | 80 | 80 | 80 | 80 |
| Z | 30 | 40 | 10 | 15 |

Table 9. The coordinates of measuring points of a line [mm]

|  | Point |  |  |
| :---: | :---: | :---: | :---: |
|  | $\# 1$ | $\# 2$ | $\# 3$ |
| X | 20 | 35 | 70 |
| Y | 85 | 85.008 | 85 |
| Z | 50 | 50 | 50 |

a)

b)

d)

c)

e)


Fig. 6. The determination of relationships: a) coaxiality of circles I and II, b) distance between circles III and IV, c) distance between planes I and II, d) angle between planes I and III, e intersection point of straight line and plane II.

## 3. TEST RESULTS

Results were grouped in four agreed upon intervals (I-correct, II-incorrect to $1 \mu \mathrm{~m}$, IIIincorrect $<1 \mu \mathrm{~m}, 2 \mu \mathrm{~m}>$ and IV-incorrect $>2 \mu \mathrm{~m}$ ). The graphical representation of the test results for circles I-IV is given in Fig. 7. The influence of the type of shape deviation on calculation results is visible. The examined software determines the equation of circle II (with three-lobing) unexpectedly well. On the other hand, the determination of the (almost ideal) circle I is a serious problem ( $21 \%$ of errors are larger than $1 \mu \mathrm{~m}$ for radius and $18 \%$ for centre). The differences between circle I and circle III is the location of point 2 . The difference is big and is should be reflected in the final results. However, we see from Fig. 4a and Fig. 4c that it has been neglected. The shift of the centres of circles I - IV leads to an error in determining the coaxiality of circles I and II and in determining the distance between circles III and IV (Fig. 8).

Similarly, large errors appear when the width $L$ of the 3 D geometrical model is calculated from the distance between planes I-III (Fig. 6c). During the actual measurement the operator does not know whether planes I and II are ideal, or whether they have any shape deviations. If planes I and II are parallel, all the tested software yields the correct result, i.e. $\mathrm{L}=100 \mathrm{~mm}$. A problem arises when there is a shape deviation in one (or both) of the planes. In this case the software must determine a substitute plane, using its (usually unpublished) fitting algorithm. The software determines substitute planes $\mathrm{PI}^{*}$ and $\mathrm{PII}^{*}$ (Fig. 9). Point \#3 plays the key role in determining plane I (Fig. 6c). It is this point that causes the shift of the whole plane $I^{*}$ in the $3 D$ space. Since the planes determined in this way are not parallel, it is not possible, in the mathematical sense, to determine the distance between them. Therefore the calculation procedure should end. However, only one software did not continue calculations after analyzing the parallelism of substitute planes PI* and PII*. As many as 13 pieces of software continued calculations (!).
a)

c)

e)

g)

b)

d)

f)

h)


Fig. 7. Measurement results: a) centre of circle I, b) radius of circle I, c) centre of circle II, d) radius of circle II, e) centre of circle III, f) g) radius of circle III, g) centre of circle IV, h) radius of circle IV.
a)

b)


Fig. 8. Relationships between circles: a) coaxiality of circles I and II, b)distance between circles III and IV.

There appears the question, then, what did the software calculate? It seems that most of the algorithms pass from one calculation option to another, without informing the operator about the change. The operator is convinced that the result is the distance between planes, but what he actually gets may be e.g. the distance between two points (Fig. 9).


Fig. 9. The determination of the distance between planes PI and PII: a) the position of substitute planes PI* and PII* obtained from calculations, b) the graphical presentation of results.

Such behaviour of the software tested made the author perform additional calculations in order to determine the used calculation algorithm Therefore the following distances were calculated (Fig.10):

- the distance between the substitute plane PI* and the centre of plane PII* (point),
- the distance between substitute plane PII* and the centre of plane PI* (point),
- the distance between the centres of substitute planes PI* and PII* (point-point).

These three measuring options appeared in seven cases of software and six cases of software apply other unknown calculation procedures. It must be noted that when we measure the distance between two non-parallel planes, we get a value which is the result of using another calculation option.

The measurement results for the angle between plane I and plane III are the most interesting (Fig. 11). Point \#3 (Fig. 6d) in plane I is essential for calculations. Because of the position of this point, the substitute plane calculated on the basis of all four points will not lie on plane YZ. Some calculation algorithms ignore point \#3 in their calculations (and use the remaining three points $1,2,3$ which lie exactly on plane YZ ). The question arises as to which result is more accurate. Is it the one which is obtained in accordance with standard DIN 32880, or the one that ignores point \#3 in the calculations? Which calculating method is better? Is it the one which performs calculations using all four measuring points, or the one which conducts an initial analysis of mutual positions of all points in the 3-D space?


Fig. 10. Block scheme of determining the distance between two planes.
a)

b)


Fig. 11. The determination of the angle between planes I-III: a) the position of the calculated plane PIII and the calculated substitute plane $\mathrm{PI}^{*}$; b) the graphical representation of results.

Other irregularities appear when we attempt to determine the common part of plane PII and a straight line (Fig. 12). This point is shifted along axis Y, which is astonishing. Since all the three points of the line lie on plane XZ, it is impossible to shift the straight line along axis Y. However, the shape deviation of plane PII causes the determination of substitute plane PII*, which is no longer parallel to plane YZ. The testing software shows that there are significant differences in calculations of the common part of the plane and the straight line. Four kinds of software yielded astonishing results for axis Y ( +1984 ; $+990 ;+2735 ;+28874$ ), which is the evidence of big irregularities in the calculating algorithms. Only three algorithms calculated the position of this point correctly.


Fig. 12. The determination of the common part of the plane and the straight line: a) the position of the calculated plane PIII and the calculated substitute plane $\mathrm{PI}^{*} ;$ b) the graphical representation of results.

The whole test has shown that we may expect a wide dispersion of the results we get in real measuring conditions, if we use the criterion of minimum measuring points. Only three algorithms have less than $10 \%$ of errors (Fig. 13a) and these algorithms improve the overall assessment of errors for the whole test (Fig. 13b).
a)

b)


Fig. 13.The graphical representation of the error values: a) for particular cases of CMM software, b) for the whole test.

From Figure 13 we might draw the conclusion that some algorithms are better than others. It must be stressed, however, that this figure shows the agreement of the results with the LMS method. Whether the LMS method is the best for all tasks, is a different issue. If we used another reference method, it could turn out that e.g. the software denoted by H , which uses other calculation algorithms, will be better than the product denoted by B. The overall conclusion of the test, however, is that with high probability we will obtain different results from the same set of input data. To illustrate the results of the software test, the software has been grouped in two classes, according to the accuracy during the test.

## Class I (software products B, F, J)

These products have some irregularities, which were visible during measurements with a small number of measuring points. Usually the errors involved were in the interval $<-1 \mu \mathrm{~m},+1 \mu \mathrm{~m}>$; disproportionately big errors also appeared, although they were very rare (the interval $>2 \mu \mathrm{~m}$ in Fig. 14). The software in class I is produced by firms which have been dealing with coordinate measurements for a long time and which have gained a lot of experience. The products of these firms are top quality. The remaining products rate Class II.

Figure 14 illustrates the differences between the two classes; $86.3 \%$ of results in class I are in the range of accurate results (in accordance with LMS), and only $7 \%$ are in the $<-1 \mu \mathrm{~m},+2 \mu \mathrm{~m}>$ error interval. In class II, on the other hand, the results are spread more
evenly in the four error intervals. The software from category II may yield relatively high errors, especially if there are few measuring points and if the points are unevenly distributed in the geometrical element.


Fig. 14. Graphical presentation of the test results.

## 4.ADDITIONAL TEST

The next step was to verify the results for an increased number of measuring points with the same shape deviations. The two programs representing those two classes ("B" and "H" software) were tested (Fig. 13a).
a)

b)

c)


Fig. 15. The example of the positioning of the measuring points in the circle with ovality: a) basic four points, b ) positioning of point $2^{*}$, c) positioning of the 32 points.

The number of measuring points was increased from 4 to $1024\left(2^{10}\right)$. The measured coordinates of the points (in the example the four points) are input into the computer calculating the suitable center and radius of the circle. However, the calculations are approximate and the user has no knowledge of the kind of applied algorithm. The error may be minimized by increasing the number of measuring points. In the test, the circle equation was pointed out with the 4 points. The positioning of those points on the circle with ovality deviation is shown in Fig. 15a. Next, the number of points was increased to 8, $16 \ldots$ and 1024. A further increase of the number is impossible because the maximum number of points may not be exceeded. Figure 15a presents how the four points are positioned on the circle. To increase the number of points, the angle between the radii R1 and R2 was divided, and the "measuring point $2^{*}$ " was placed in the crossing of the bisector and the circle (Fig. 15b). The rest of the points were positioned in the same way (Fig. 15c). Figure 16 presents the results as a circle equation pointed out with chosen CMM programs for the increased number of points.
a)

b)

Fig. 16. The influence of the number of measuring points on the values of: a) centres I-IV, b) radiuses I-IV.


Fig. 17. Graphical presentation of the test results.
Figure 17 illustrates the differences between the two CMMs software with 4 and 1024 measuring points. It is visible that the extension of the number of measuring points to 1024 in a fundamental manner changes the results of the test. The resolution became smaller by the difference in performance between CMMs B and H software. A similar result appears in remaining CMMs software reckoned of being class II. As a resume, to execute the measurement with the scanning method practically one obtains approximate results with the adaptation of the LMS method or its modification.

## 5. CONCLUSIONS

The analysis of accuracy of calculation algorithms of CMMs, conducted in the 90s, showed a high level of errors in the obtained measurement results [1]. The problem of CMM accuracy has been noticed again fairly recently by a number of researchers [2-12]. The results of the test conducted by the author on currently-used calculating algorithms show that the problem of CMM accuracy is still unsolved. The CMM software may significantly influence the accuracy of the whole measurement system. Until now, the manufacturers of CMM have been giving the MPE $_{E}$ (maximum permissible error of indication of a CMM for size measurement) [14], which refers only to the CMM. The computer and the software, or the calculation algorithm, which are now intrinsic parts of the CMM measuring system, have been treated as factors which do not introduce any errors. However, on the basis of the test which involved 14 different algorithms it can be said that the computer and software is the source of significant errors, which in unfavourable conditions can be much higher than the $\mathrm{MPE}_{\mathrm{E}}$ given by the manufacturers.

The calculation algorithms used in software offered by different manufacturers are not identical. In many cases the algorithm used is not the classical Gaussian algorithm (LMS). Manufactures introduce modifications of this algorithm; however, they do not inform the
users about them. It must be stressed that there are no known fitting algorithms which are ideal, particularly for measuring geometrical elements with shape deviations. Therefore the tendency to modify the basic fitting algorithms is understandable. However, some modifications are better suited to detect some kinds of deviations and are absolutely useless for detecting others. It seems that we cannot describe all geometrical elements with various shape deviations with one fitting function; we will always obtain an approximation only. Therefore it seems necessary to work out new fitting algorithms using a specified location and number of measurement points, suited to particular shape deviations.

Some software offers more than one method for finding the circle equation. The centre and the radius of a circle may be determined using the following methods: LMS, MZ, MIC and MCC. In this case, however, the operator must decide which method to choose and what criteria to use in making the decision. For a relatively small number of measuring points it is difficult to choose the appropriate fitting method, since fitting methods become stable only for a large number of measuring points. The minimal numbers applied today do not ensure it; they must be increased considerably and related to the kind of the shape deviation. The shape deviations of the real details are complicated and their description is difficult, which means the need of application of a large number of measuring points. The accuracy of the measurement also depends on the positioning of those points.

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